Solvay Brussels School
Economics & Management

ULB

Risk Management and Governance
Reminder on Derivatives

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Quick presentation made in class of the main categories of derivatives and short reminder…

Types

- Commitments (Forwards & Futures, Swaps) ~ Linear products

Coffee Forward

40	80
(+15)	(-25)
55	55

40

Cash settlement
Physical delivery

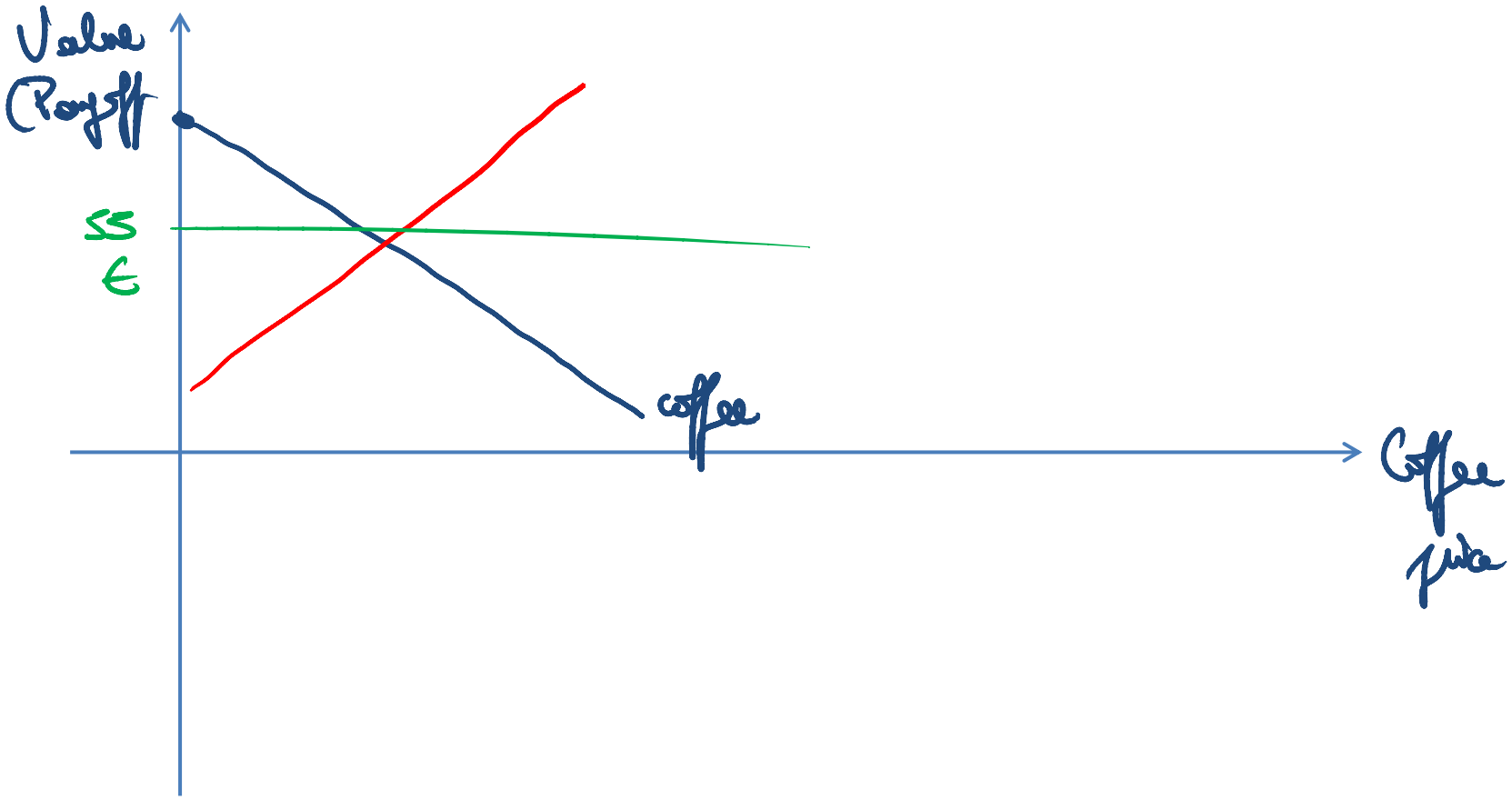
- Insurances (optional products)

you would have to choose "flexible instruments"

Options: Call, Put
+ many other "exotics" and combinations.

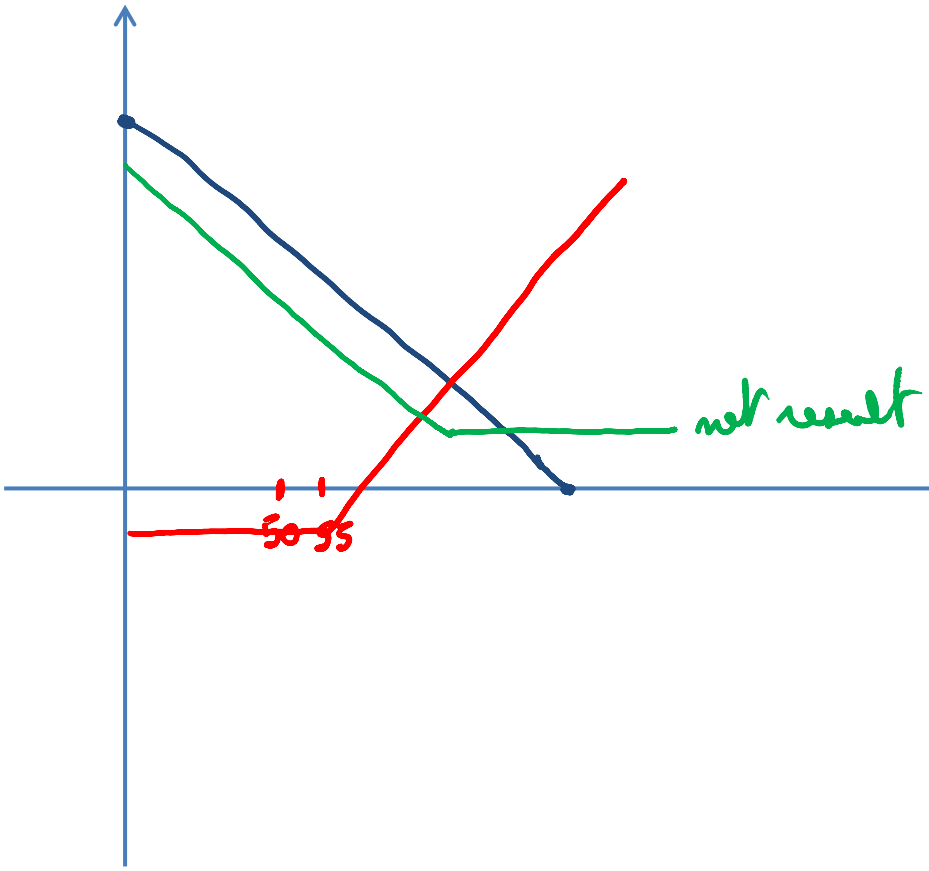
Ex: Nestlé buying coffee

Profiles (1)



Profiles (2)

What if with options (?)



2 choices

forwards
t=0 nothing
T difference

options
t=0 a little thing paid
T only ⊕ differences.

How do we value them?

- Commitments

Lucien says, it is just about expectations & discount factors.

DO-IT YOURSELF

$F_T = S_0 \times (1 + r_{0,T})^T + \text{storage costs} - \text{"convenience"}$

- Insurances

Options, so we need to assess probabilities of exercise

- MC simulations
- binomial trees
- closed-form solutions (formula): B&Sch for standard options.
  - better for sensitivity analysis GREEKS.

Some theoretical elements, as a review, and for some of you, seen in the course of Derivatives more in detail. We will need to have these elements for us in our course, but focusing less on valuation than on uses and hedging with them. We will get back to them when needed.

Financial markets

- Exchange traded
  - Traditionally exchanges have used the open-outcry system, but increasingly they are switching to electronic trading
  - Contracts are standard; there is virtually no credit risk
  - Ex:
    - ✓ NYSE: stock trading
    - ✓ CBOT + CBOE: trading of futures and options
  - The exchange
    - ✓ Clearinghouse
    - ✓ Clearing members
- Over-the-counter (OTC)
  - A computer- and telephone-linked network of dealers at financial institutions, corporations, and fund managers
  - Contracts can be non-standard; there is some small amount of credit risk
  - Phone conversations are taped

The products

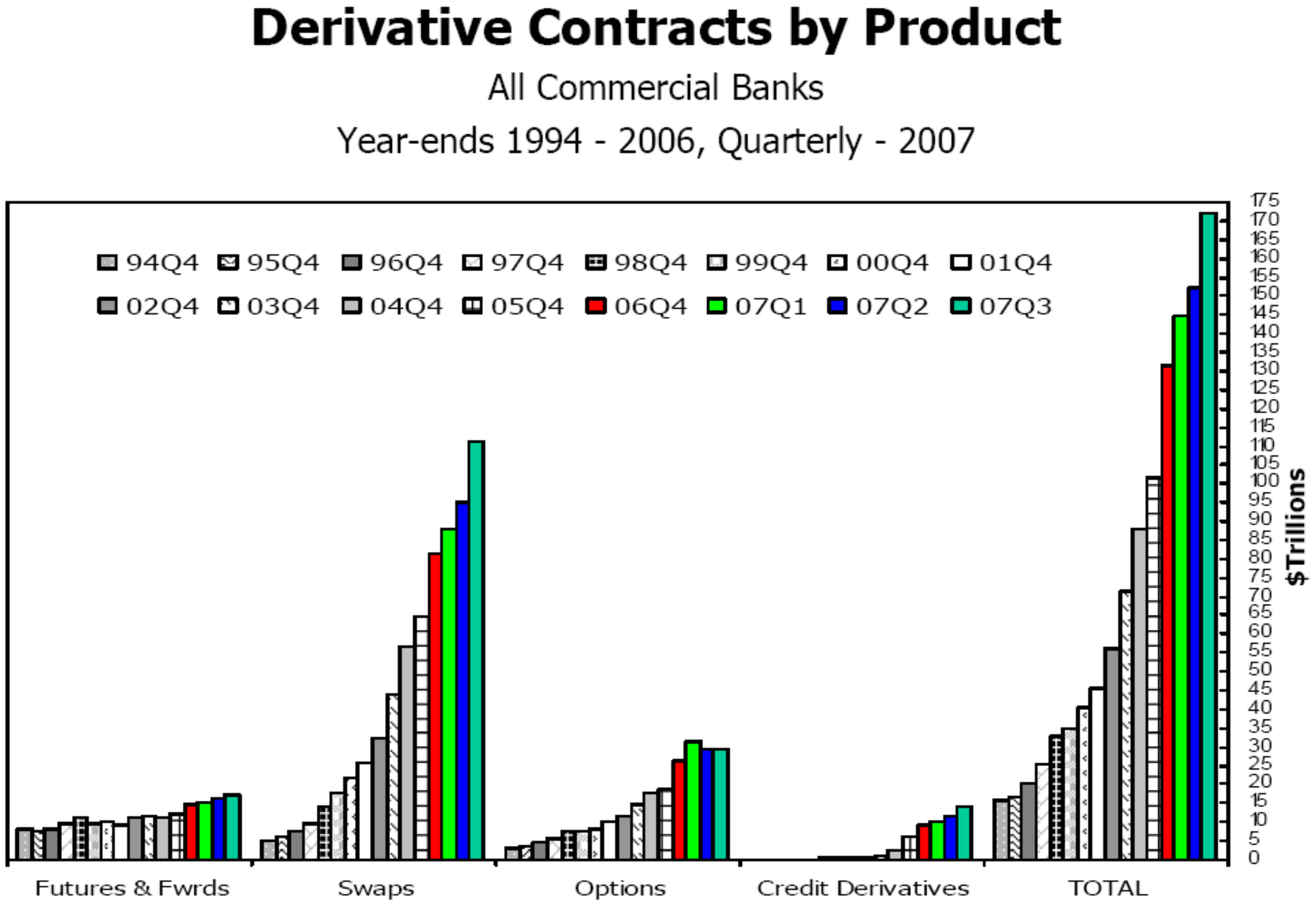
- Plain-vanilla
  - Long/short positions
  - Forwards & Futures
  - Swaps
  - Standard options
- Exotics
  - Asian options
  - Basket options
  - Binary or digital options
  - Compound options
  - Barrier options
  - Lookback options
  - ...
- Structured products

The agents

- Hedgers
- Speculators
- Arbitrageurs

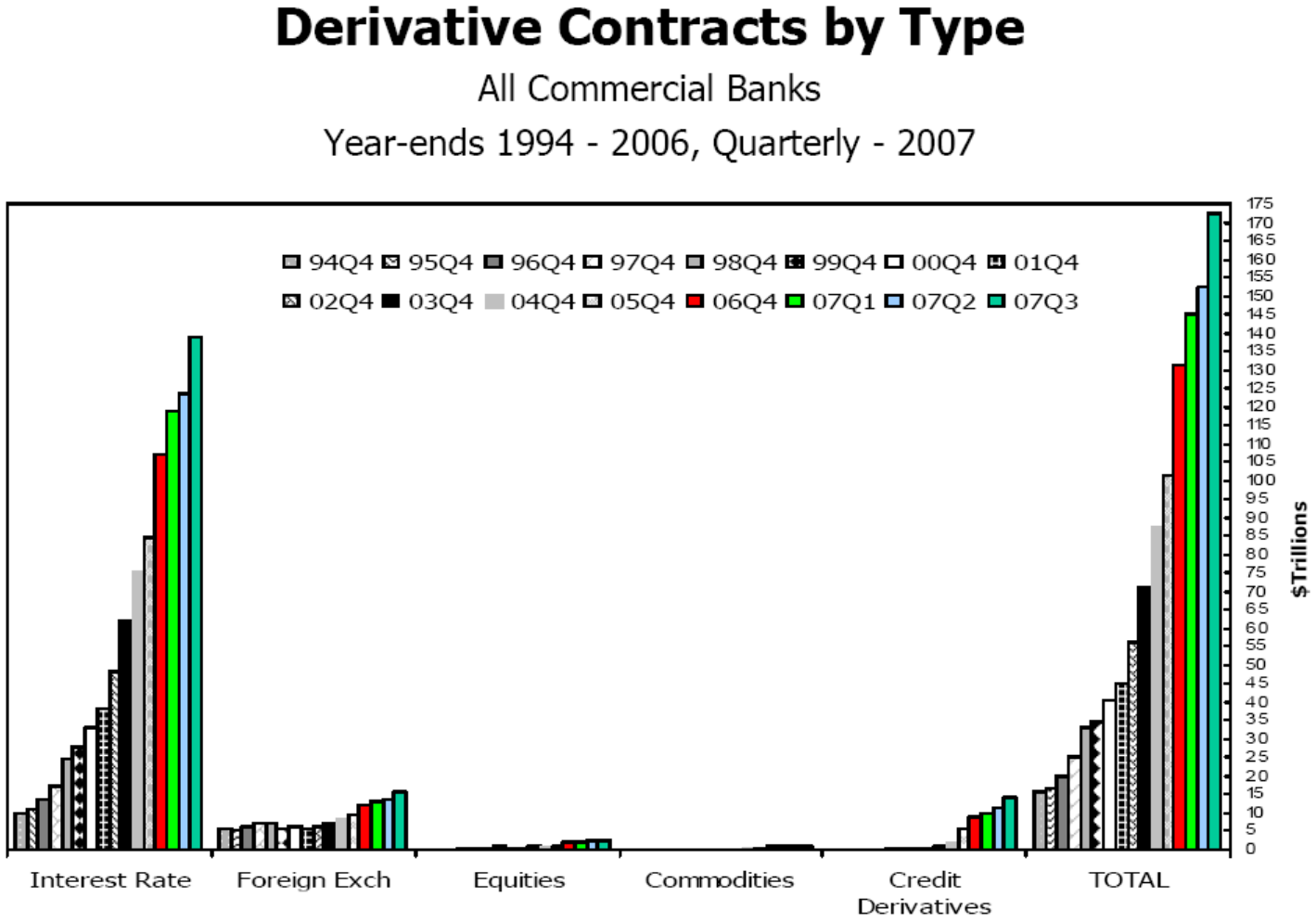
Some of the largest trading losses in derivatives have occurred because individuals who had a mandate to be hedgers or arbitrageurs switched to being speculators

The products (2)



Source: Quarterly Derivatives Fact Sheet, The Office of the Comptroller of the Currency (OCC), Administrator of the National Banks, USA.

The products (2)



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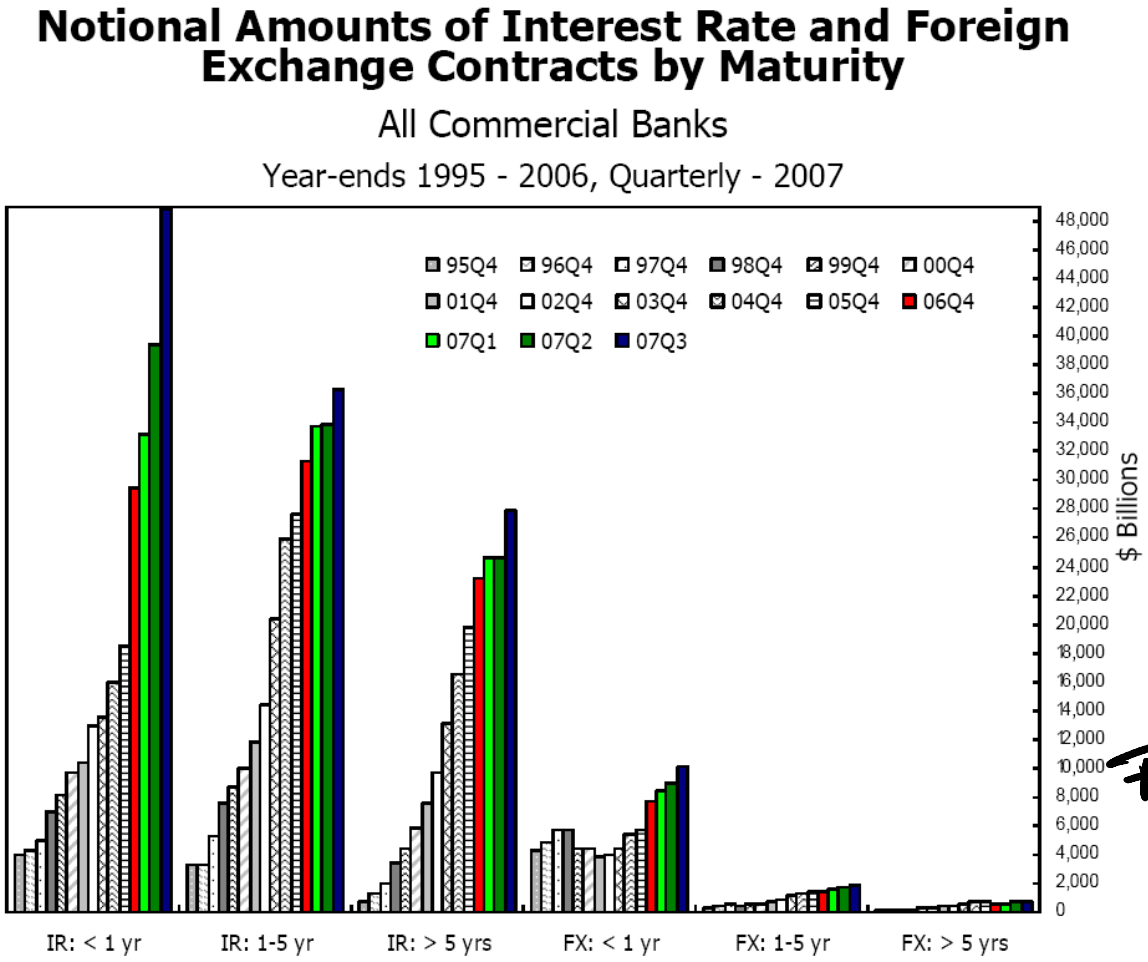
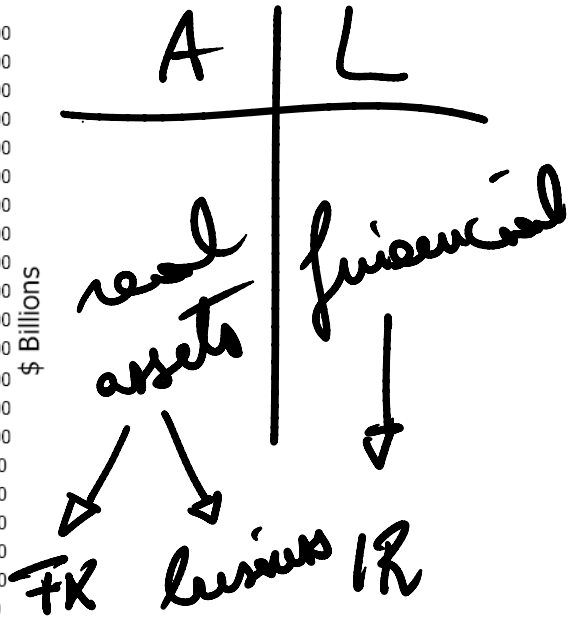
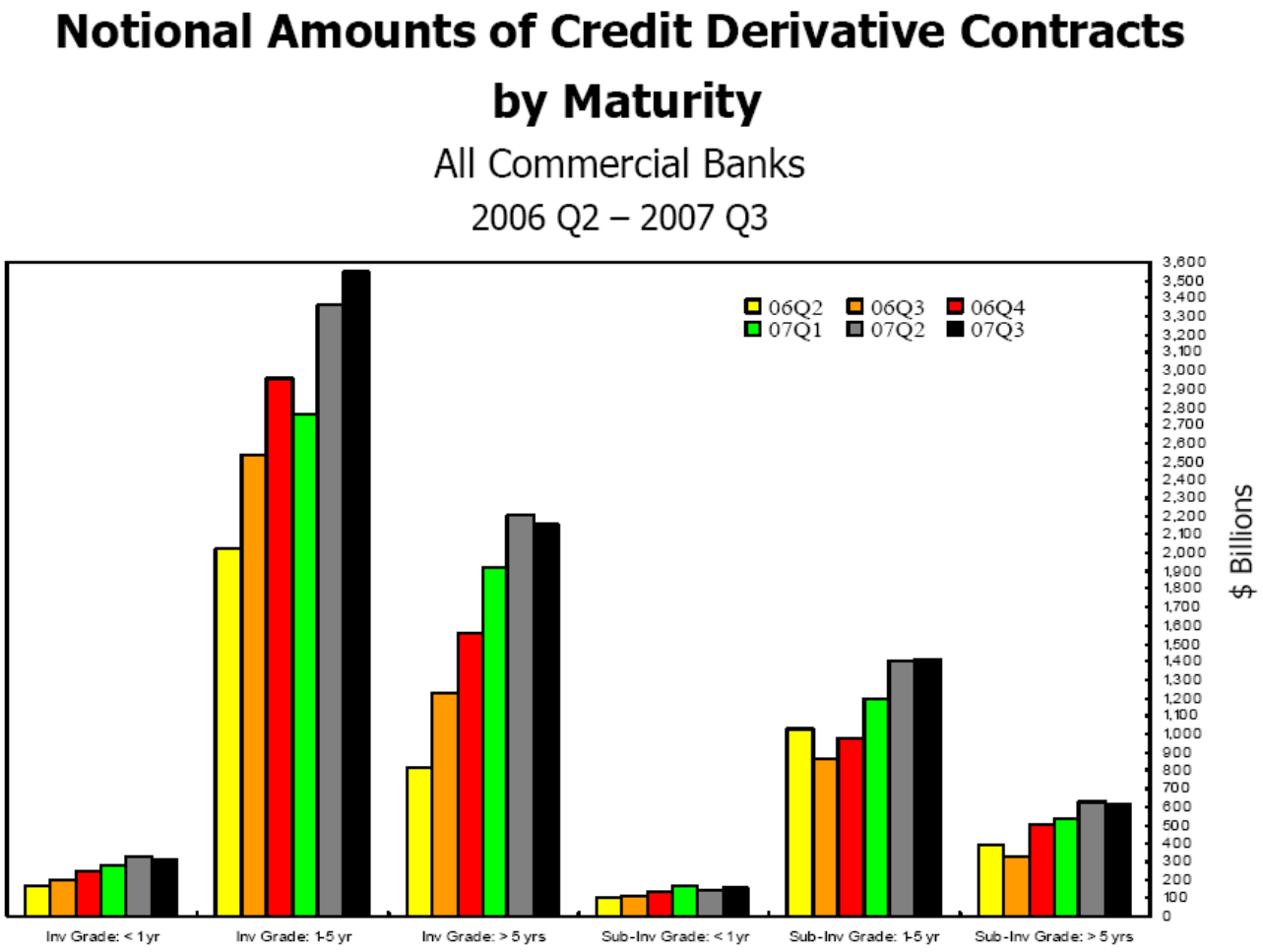
The products (3)





Source: Quarterly Derivatives Fact Sheet, The Office of the Comptroller of the Currency (OCC), Administrator of the National Banks, USA.

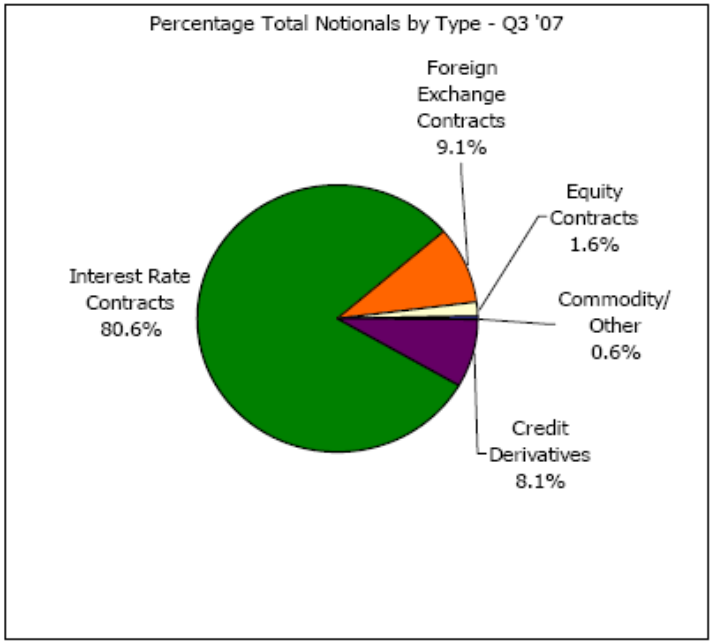
The products (4)

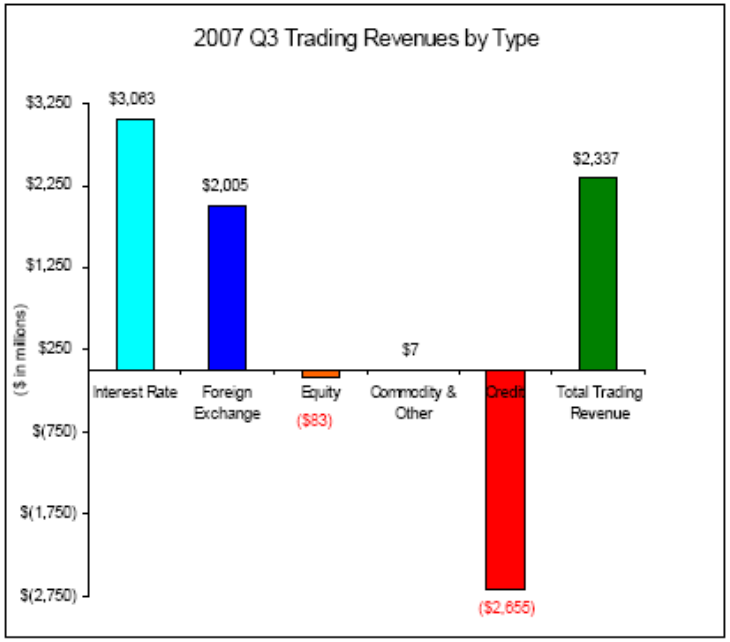


Source: Quarterly Derivatives Fact Sheet, The Office of the Comptroller of the Currency (OCC), Administrator of the National Banks, USA.

The revenues in 2007

Of the trading revenue components, interest rate revenues were the strongest, increasing 3%, or $102 million, to a record $3.1 billion. Foreign exchange revenues were also notable at $2.0 billion, a 59% increase from the previous quarter. The credit market turmoil in the third quarter caused revenues from credit trading to fall $3.5 billion to a loss of $2.7 billion. The losses in credit trading resulted from the sharp increase in credit spreads that occurred in the third quarter, creating a difficult environment for trading and hedging, particularly against correlation risks. Overall client demand was healthy as bank clients engaged in derivatives contracts to offset risks arising in highly volatile market conditions.





Data Source: Call Reports. Note: Beginning 1Q07, credit exposures are broken out as a separate revenue category.

Source: Quarterly Derivatives Fact Sheet, The Office of the Comptroller of the Currency (OCC), Administrator of the National Banks, USA.

BIS Worldwide OTC statistics

Table 19: Amounts outstanding of over-the-counter (OTC) derivatives

By risk category and instrument

In billions of US dollars

	Notional amounts outstanding					Gross market values				
Risk Category / Instrument	Jun 2005	Dec 2005	Jun 2006	Dec 2006	Jun 2007	Jun 2005	Dec 2005	Jun 2006	Dec 2006	Jun 2007
Total contracts	**281,493**	**297,670**	**369,507**	**414,290**	**516,407**	**10,605**	**9,749**	**9,936**	**9,682**	**11,140**
Foreign exchange contracts	**31,081**	**31,364**	**38,091**	**40,239**	**48,620**	**1,141**	**997**	**1,134**	**1,264**	**1,343**
Forwards and forex swaps	15,801	15,873	19,395	19,870	24,526	464	406	435	468	492
Currency swaps	8,236	8,504	9,669	10,767	12,291	549	453	533	599	617
Options	7,045	6,987	9,027	9,602	11,804	129	138	166	196	235
Interest rate contracts	**204,795**	**211,970**	**261,960**	**291,115**	**346,937**	**6,699**	**5,397**	**5,435**	**4,820**	**6,057**
Forward rate agreements	13,973	14,269	18,117	18,668	22,809	31	22	25	32	43
Interest rate swaps	163,749	169,106	207,042	229,241	271,853	6,077	4,778	4,831	4,157	5,315
Options	27,072	28,596	36,800	43,206	52,275	592	597	579	631	700
Equity-linked contracts	**4,551**	**5,793**	**6,782**	**7,488**	**9,202**	**382**	**582**	**671**	**853**	**1,116**
Forwards and swaps	1,086	1,177	1,430	1,767	2,599	88	112	147	166	240
Options	3,464	4,617	5,351	5,720	6,603	294	470	523	686	876
Commodity contracts	**2,940**	**5,434**	**6,394**	**7,115**	**7,567**	**376**	**871**	**718**	**667**	**670**
Gold	288	334	456	640	426	24	51	77	56	47
Other commodities	2,652	5,100	5,938	6,475	7,141	351	820	641	611	623
Forwards and swaps	1,748	1,909	2,188	2,813	3,447					
Options	904	3,191	3,750	3,663	3,694					
Credit default swaps	**10,211**	**13,908**	**20,352**	**28,650**	**42,580**	**188**	**243**	**294**	**470**	**721**
Single-name instruments	7,310	10,432	13,873	17,879	24,239	136	171	186	278	406
Multi-name instruments	2,901	3,476	6,479	10,771	18,341	52	71	109	192	315
Unallocated	**27,915**	**29,199**	**35,928**	**39,682**	**61,501**	**1,818**	**1,659**	**1,683**	**1,608**	**1,233**
Memorandum Item:										
Gross Credit Exposure						1,897	1,900	2,029	2,034	2,669

Source: BIS market statistics (http://www.bis.org/statistics), 2007

BIS Worldwide exchange-traded statistics (1)

Table 23A: Derivative financial instruments traded on organised exchanges

By instrument and location

Notional principal in billions of US dollars

	Amounts outstanding				Turnover					
Instrument / location	Dec 2005	Dec 2006	Jun 2007	Sep 2007	2005	2006	Q4 2006	Q1 2007	Q2 2007	Q3 2007
Futures										
All markets	**21,600.3**	**25,683.0**	**31,676.9**	**28,638.5**	**1,004,378.7**	**1,261,649.2**	**310,353.9**	**378,225.4**	**374,721.2**	**457,698.2**
Interest rate	20,708.8	24,476.2	30,147.8	27,178.6	939,590.1	1,169,300.4	285,309.9	346,198.5	339,546.4	413,551.0
Currency	107.6	161.4	201.8	189.1	11,125.8	15,154.0	4,291.3	4,701.5	4,704.1	5,592.0
Equity index	784.0	1,045.3	1,327.2	1,270.8	53,662.8	77,194.9	20,752.7	27,325.5	30,470.7	38,555.2
North America	**12,326.8**	**13,741.9**	**15,937.1**	**14,711.8**	**564,237.1**	**713,937.6**	**175,221.8**	**209,852.4**	**201,317.7**	**253,078.7**
Interest rate	11,855.2	13,077.0	15,153.1	13,973.3	529,120.9	667,386.0	162,514.4	195,063.9	185,593.5	232,389.9
Currency	90.7	136.4	155.6	120.0	10,258.4	13,684.8	3,901.3	4,175.8	4,063.6	4,855.7
Equity index	380.8	528.5	628.3	618.6	24,857.9	32,866.8	8,806.2	10,612.7	11,660.6	15,833.1
Europe	**6,275.4**	**8,150.3**	**10,525.1**	**9,349.2**	**380,538.4**	**455,715.3**	**111,458.2**	**139,779.1**	**140,768.0**	**169,887.4**
Interest rate	6,050.5	7,801.7	10,049.3	8,921.4	362,066.3	427,979.7	103,675.3	128,691.4	128,565.2	155,566.5
Currency	2.4	1.8	3.3	5.1	36.6	45.7	12.9	17.6	15.4	42.9
Equity index	222.5	346.8	472.5	422.7	18,435.5	27,689.9	7,770.0	11,070.0	12,187.5	14,277.9
Asia and Pacific	**2,685.5**	**3,369.0**	**4,658.7**	**3,994.1**	**51,726.2**	**81,356.8**	**20,711.0**	**24,863.4**	**27,509.8**	**29,929.0**
Interest rate	2,509.8	3,210.7	4,448.8	3,766.5	41,666.7	65,713.2	16,862.9	19,730.6	21,569.3	22,229.6
Currency	4.3	8.1	17.9	39.7	133.7	162.9	47.2	44.8	46.1	82.4
Equity index	171.4	150.1	192.0	187.9	9,925.9	15,480.7	3,801.0	5,088.0	5,894.4	7,617.0
Other Markets	**312.7**	**421.7**	**556.0**	**583.4**	**7,877.0**	**10,639.5**	**2,962.8**	**3,730.6**	**5,125.8**	**4,803.2**
Interest rate	293.2	386.8	496.6	517.5	6,736.3	8,221.5	2,257.3	2,712.5	3,818.4	3,365.0
Currency	10.2	15.0	24.9	24.4	697.1	1,260.6	330.0	463.3	579.1	611.0
Equity index	9.3	20.0	34.4	41.6	443.5	1,157.4	375.5	554.8	728.3	827.2

Source: BIS market statistics (http://www.bis.org/statistics), 2007

BIS Worldwide exchange-traded statistics (2)

Options										
All markets	**36,188.1**	**44,760.4**	**65,006.7**	**66,244.8**	**402,582.1**	**546,471.9**	**120,597.6**	**153,948.0**	**161,734.3**	**223,462.3**
Interest rate	31,588.3	38,116.5	55,987.1	56,453.8	328,778.9	446,022.0	95,790.7	120,490.3	123,125.8	180,078.6
Currency	66.1	78.6	101.2	120.9	944.0	1,119.9	317.7	376.9	517.9	588.7
Equity index	4,533.7	6,565.3	8,918.4	9,670.1	72,859.3	99,330.0	24,489.2	33,080.8	38,090.6	42,795.1
North America	**24,058.4**	**28,809.4**	**41,995.5**	**37,620.9**	**254,496.4**	**361,397.7**	**78,505.9**	**94,778.5**	**101,709.1**	**128,430.0**
Interest rate	21,255.4	24,844.4	36,823.9	32,066.2	229,976.4	326,268.9	69,238.2	83,209.9	88,755.7	114,188.2
Currency	28.3	32.6	45.3	52.1	449.0	453.1	120.8	128.2	140.4	155.5
Equity index	2,774.8	3,932.5	5,126.3	5,502.6	24,070.9	34,675.6	9,147.0	11,440.4	12,813.0	14,086.3
Europe	**11,697.8**	**15,066.5**	**21,752.9**	**27,183.9**	**105,910.3**	**127,015.9**	**29,294.0**	**41,516.3**	**38,804.9**	**70,731.0**
Interest rate	10,235.7	12,702.2	18,312.1	23,377.5	96,704.2	113,554.5	25,230.7	35,610.1	32,413.8	63,635.8
Currency	0.6	0.7	0.8	0.8	7.8	10.4	4.3	1.7	1.5	1.8
Equity index	1,461.5	2,363.6	3,440.1	3,805.6	9,198.2	13,451.0	4,058.9	5,904.5	6,389.6	7,093.4
Asia and Pacific	**319.0**	**680.7**	**990.5**	**1,098.9**	**40,312.1**	**55,061.5**	**11,970.7**	**16,551.1**	**19,923.8**	**22,781.1**
Interest rate	67.4	459.7	691.6	803.4	1,947.8	5,593.7	1,125.6	1,426.7	1,723.4	1,948.5
Currency	–	–	–	–	–	–	–	–	–	–
Equity index	251.6	221.0	299.0	295.4	38,364.4	49,467.8	10,845.2	15,124.5	18,200.4	20,832.6
Other Markets	**112.9**	**203.8**	**267.7**	**341.2**	**1,863.2**	**2,996.8**	**827.0**	**1,102.0**	**1,296.6**	**1,520.3**
Interest rate	29.8	110.2	159.6	206.6	150.4	604.9	196.2	243.6	233.0	306.1
Currency	37.2	45.4	55.1	68.1	487.1	656.4	192.6	247.1	376.0	431.4
Equity index	45.8	48.2	53.1	66.5	1,225.7	1,735.6	438.2	611.4	687.6	782.8

Source: BIS market statistics (http://www.bis.org/statistics), 2007

Short-selling

- Short selling involves selling securities you do not own
- Your broker borrows the securities from another client and sells them in the market in the usual way
- At some stage you must buy the securities back so they can be replaced in the account of the client
- You must pay dividends and other benefits the owner of the securities receives

Forward contract

- Contract/agreement whereby parties are committed:
  - to buy (sell)
  - an underlying asset
  - at some future date (maturity)
  - at a delivery price (forward price) set in advance
- Forward contracts trade in the over-the-counter market
- They are particularly popular on currencies and interest rates
- Trading:
  - Buying forward = "LONG" position
  - Selling forward = "SHORT" position
- Cash-flows:
  - t_0 : No cash flow
  - T : Obligation to transact
- Market
  - Allows to liquidate position by taking a reverse one
- Particularities:
  - Cash settlement vs. physical delivery

Cash flows

- Notations
  - S_T Price of underlying asset at maturity
  - F_t Forward price (delivery price) set at time $t < T$

Position	Initiation	Maturity T
Long	0	$S_T - F_t$
Short	0	$F_t - S_T$

- Initial cash flow = 0 : delivery price equals forward price.
- Credit risk during the whole life of forward contract.
- Locking-in the result before maturity
  - At time t_1: Enter a new forward contract in opposite direction.
    - ✓ Ex : long forward at forward price F_1
  - At time t_2 ($< T$): Short forward at new forward price F_2
  - Gain/loss at maturity :
    - ✓ $(S_T - F_1) + (F_2 - S_T) = F_2 - F_1$ no remaining uncertainty

Definition

- Traded standardized version of forward
  - » Institutionalized forward contract with daily settlement of gains and losses
  - » Standardized:
    - ✓ Maturity
    - ✓ Face value of contract
    - ✓ Quality
- Traded on an organized exchange
  - » Clearing house
- Daily settlement of gains and losses (Marked to market)
  - » In a forward contract:
    - ✓ Buyer and seller face each other during the life of the contract
    - ✓ Gains and losses are realized when the contract expires
    - ✓ Credit risk
      - BUYER ⇔ SELLER
  - » In a futures contract
    - ✓ Gains and losses are realized daily (Marking to market)
    - ✓ The clearinghouse garantees contract performance : steps in to take a position opposite each party
      - BUYER ⇔ CH ⇔ SELLER

Margin requirements

- Elements
  - » INITIAL MARGIN : deposit to put up in a margin account by a person entering a futures contract
  - » MAINTENANCE MARGIN : minimum level of the margin account
  - » MARKING TO MARKET :

	CASH FLOWS	
Change in futures price	LONG(buyer)	SHORT(seller)
$F_{t+1} - F_t$	+ Size $\times$ $(F_{t+1} - F_t)$	– Size $\times$ $(F_{t+1} - F_t)$

- Equivalent to writing a new futures contract every day at the new futures price
  (Remember how to close of position on a forward)
- Note: timing of cash flows different

Example: Barings

- Long position on 20,000 Nikkei 225 Futures
- 1 index pt = Yen 1,000 = $ 10
  - » If Nikkei 225 = 20,000
  - » Size of contract = $ 200,000 $\Rightarrow$ position =$ 4,000 mio
Date	Nikkei 225	
30.12.94	19,723	
25.02.95	17,473	Δ F = - 2,250

- Loss $= \Delta F \times \$/pt \times \#\ contracts$

  $= (-2{,}250) \times (\$\ 10) \times (20{,}000) = \$\ 450{,}000{,}000$

Pricing → Key ideas

- DECOMPOSITION
  - » Two different ways to own a unit of the underlying asset at maturity:
    1. Buy spot (SPOT PRICE: S_t) and borrow
       => Interest and inventory costs
    2. Buy forward (AT FORWARD PRICE F_t)
- By the AOA, in perfect markets, no free lunch.
  - » The 2 methods should cost the same.

Pricing → Decomposition (no income)

- Notations
  - ✓ F_t : Forward price set at time t
  - ✓ K : Delivery price
  - ✓ f_t : Value of forward contract (When contract initiated : $K = F \Leftrightarrow f = 0$)
  - ✓ T : Maturity
- Decomposition : compare the two equivalent following strategies

Position	Cash flow t	Cash flow T
Long forward	0	$S_T - F_t$
Buy spot	$- S_t$	$+ S_T$
& borrow	$+ S_t$	$- F_t$

- Synthetic forward contract:
  - » Long position on the underlying asset
  - » Short position on a zero coupon with face value = F_t

Final Pricing

- No arbitrage opportunity : in a perfect capital market, value of forward contract = value of synthetic forward contract
  - » $f_t = S_t - K \times d(t,T)t = S_t - \text{PV}(K)$
  - » With continuously compounded interest rate:
    - ✓ $f_t = S_t - Ke^{-r(T-t)}$
- Forward price : Delivery price such that $f_t = 0$
  - » $F_t = S_t / d(t,T) = \text{FV}(S_t)$
  - » With continuously compounded interest rate:
    - ✓ $F_t = S_t\, e^{r(T-t)}$
- Also:

  $F_t = S_t\, e^{r(T-t)}$

  $\Rightarrow f_t = (F_t - K)\, e^{-r(T-t)}$
  - » $f_t > 0 \Leftrightarrow F_t > K$
  - » $f_t = 0 \Leftrightarrow F_t = K$
  - » $f_t < 0 \Leftrightarrow F_t < K$

Arbitrage 1: Cash and carry

- If forward price quoted on the market (K) is greater than its theoretical value (F_t)
  - » $K > F_t = S_t \exp[r(T-t)] \Leftrightarrow PV(K) > S_t$
- As: $f_t = (F_t - K) \times d(t,T)$, $f_t < 0$
  - » The "true value" of the contract is negative.
  - » But the market price for the contract is 0.
  - » Hence, the contract is overvalued by the market.
- $\Rightarrow$ Cash-and-carry arbitrage :
  - » Sell overvalued forward
  - » Buy synthetic forward: buy spot and borrow

Cash and carry: Arbitrage table

Cash flows	CF_t	CF_T
(1) Buy spot	$- S_t$	$+ S_T$
(2) Borrow	$+PV(K)$	$-K$
(3) Sell fwd	0	$K-S_T$
TOTAL	$-S_t + PV(K) > 0$	0

- Conclusion:
  - To avoid arbitrage, $CF_t = -S_t + Ke^{-r(T-t)} \leq 0$
- $K \leq S_t\, e^{r(T-t)}$, $f_t \leq S_t - Ke^{-r(T-t)}$
- Note:
  - the arbitrage could be designed to obtain a future profit at time T by borrowing S_t
- $CF_t = 0$ and $CF_T = K - S_t\, er(T-t) > 0$

Arbitrage 2: Reverse cash and carry

- If forward price quoted on the market (K) is less than its theoretical value (F_t)
  - » $K < F_t = S_t \, exp[r(T-t)] \Leftrightarrow PV(K) < S_t$
- As: $f_t = (F_t - K) \times d(t,T)$, $f_t > 0$
  - » The "true value" of the contract is positive.
  - » But the market price for the contract is 0.
  - » Hence, the contract is undervalued by the market.
- $\Rightarrow$ reverse cash-and-carry arbitrage :
  - » Buy undervalued forward (futures)
  - » Sell synthetic forward (futures)

Reverse cash and carry : arbitrage table (with future profit)

Cash flows	CF_t	CF_T
(1) Sell spot	$+S_t$	$- S_T$
(2) Invest	$- S_t$	$+ S_t e^{r(T-t)}$
(3) Buy forward	0	$+ (S_T-K)$
TOTAL	0	$S_t e^{r(T-t)} - K > 0$

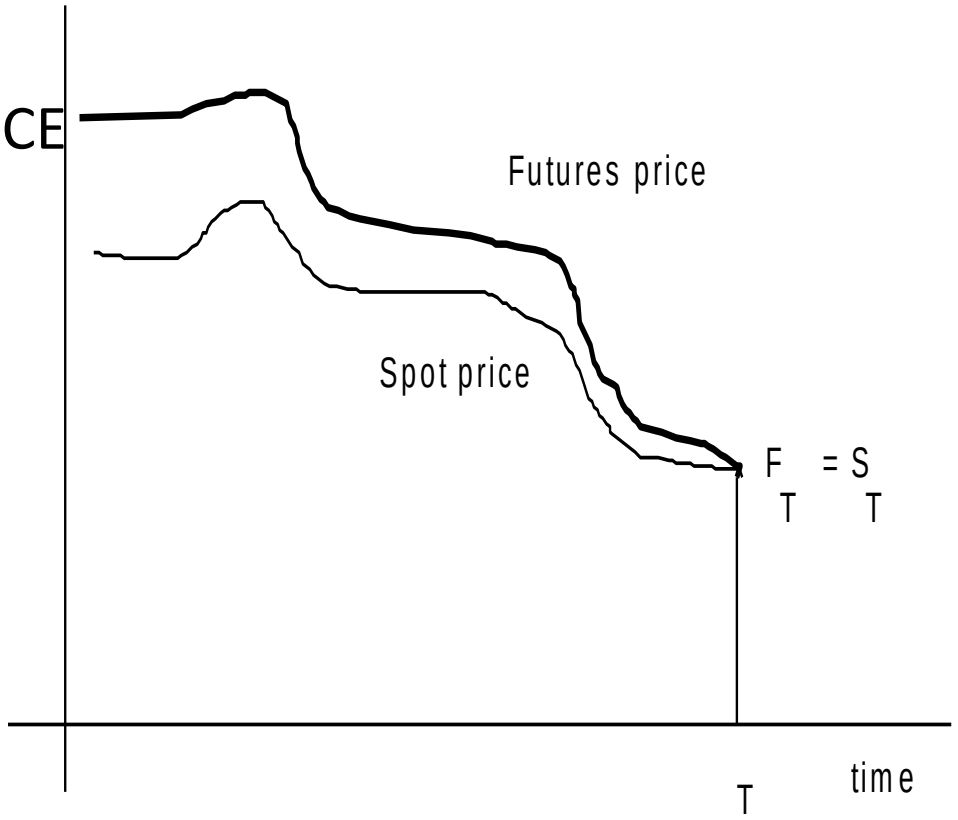
- To avoid arbitrage, $S_t e^{r(T-t)} - K \leq 0$
- $K \geq S_t e^{r(T-t)} = F_t$, $f_t \geq t - Ke^{-r(T-t)}$
- Note:
  - the arbitrage could be designed to obtain an immediate profit at time *t* by investing PV(*K*)
- $CF_t = S_t - PV(K) > 0$ and $CF_T = 0$

Equilibrium

- If both arbitrage are possible:
  - $f_t = S_t - Ke^{-r(T-t)}$
- When the contract is initiated:
  - $K = F \;\&\; f = 0$
  - $0 = S_t - F_t\, e^{-r(T-t)}$
  - $F_t = S_t\, e^{r(T-t)}$

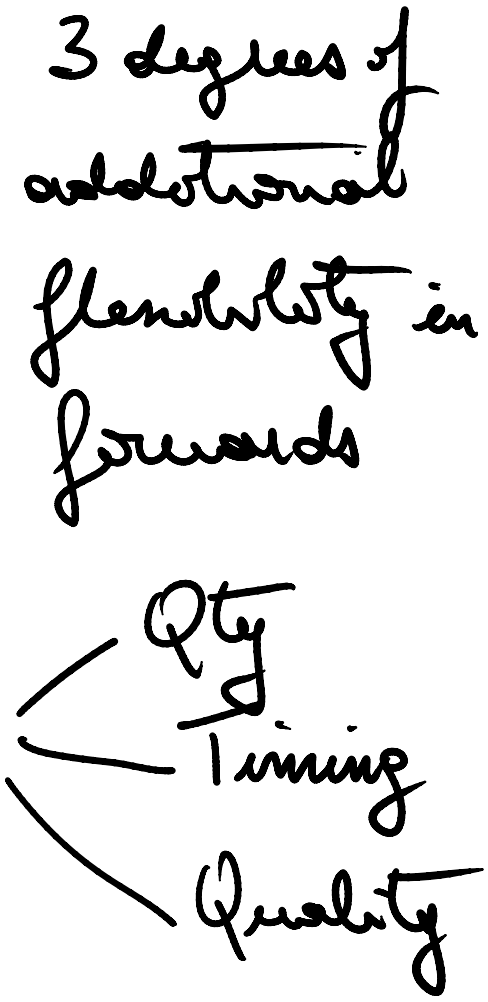
Definition

- DEFINITION :
  SPOT PRICE - FUTURES PRICE
  - » $b_t = S_t - F_t$
- Depends on:
  - » level of interest rate
  - » Time to maturity
    (↓ as maturity ↓)



Basis risk: Numerical Example r = 10.00%

Mois	T-t	S_t	F_t	$BASE$	f_t
0	1.000	100.00	110.52	-10.52	0.00
1	0.917	104.42	114.44	-10.02	3.58
2	0.833	109.15	118.63	-9.49	7.47
3	0.750	111.63	120.32	-8.69	9.10
4	0.667	111.75	119.46	-7.70	8.36
5	0.583	111.09	117.76	-6.67	6.83
6	0.500	106.63	112.10	-5.47	1.51
7	0.417	105.06	109.53	-4.47	-0.95
8	0.333	107.33	110.96	-3.64	0.43
9	0.250	106.68	109.38	-2.70	-1.11
10	0.167	103.50	105.24	-1.74	-5.19
11	0.083	101.34	102.19	-0.85	-8.26
12	0.000	101.35	101.35	0.00	-9.16



Extensions: Known dividend yield

- q : dividend yield p.a. paid continuously
- $F = [\, e^{-q(T-t)} S_t \,] \, e^{r(T-t)} = S_t \, e^{(r-q)(T-t)}$

- Examples:
  - » Forward contract on a Stock Index
    - ✓ r = interest rate
    - ✓ q = dividend yield
- Foreign exchange forward contract:
    - ✓ r = domestic interest rate (continuously compounded)
    - ✓ q = foreign interest rate (continuously compounded)

Extensions: Commodities

- I = - PV of storage cost (negative income)

- q = - convenience yield

Valuation of futures contracts

- If the interest rate is non stochastic, futures prices and forward prices are identical
- NOT INTUITIVELY OBVIOUS:
  - Total gain or loss equal for forward and futures
  - but timing is different
    - ✓ Forward : at maturity
    - ✓ Futures : daily

Forward price & expected future price

- Is F an unbiased estimate of $E(S_T)$?
  - » $F < E(S_T)$ Normal backwardation
  - » $F > E(S_T)$ Contango

- $F = E(S_T)\, e^{(r-k)\,(T-t)}$
  - » *If* $k = r$ $F = E(S_T)$
  - » *If* $k > r$ $F < E(S_T)$
  - » *If* $k < r$ $F > E(S_T)$

Swaps

- Contract/agreement whereby parties are committed:
  - To exchange cash flows
  - At future dates
  - according to certain specified rules
- Two most common contracts:
  - Interest rate swaps (IRS)
  - Currency swaps (CS)
- Ex: Uses of an IRS
  - Converting a liability from
    - ✓ fixed rate to floating rate
    - ✓ floating rate to fixed rate
  - Converting an investment from
    - ✓ fixed rate to floating rate
    - ✓ floating rate to fixed rate

Definition of plain vanilla interest rate swap

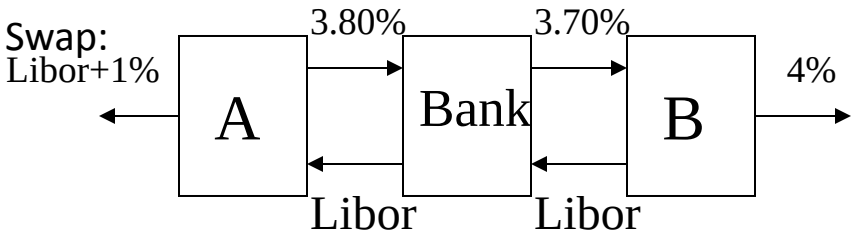
- Contract by which
  - Buyer (long) committed to pay fixed rate R
  - Seller (short) committed to pay variable r (Ex:LIBOR)

  - on notional amount M
  - No exchange of principal
  - at future dates set in advance
  - $t + \Delta t, t + 2\,\Delta t, t + 3\Delta t , t+ 4\,\Delta t, \ldots$

- Most common swap : 6-month LIBOR

Interest Rate Swap Example

	Objective	Borrowing conditions	
		Fix	Var
A	Fix	5%	Libor + 1%
B	Var	4%	Libor+ 0.5%



Gains for each company

	A	B
Outflow	Libor+1%	4%
	3.80%	Libor
Inflow	Libor	3.70%
Total	4.80%	Libor+0.3%
Saving	0.20%	0.20%

A free lunch ?

Other example

- Ex: an agreement to receive 6-month LIBOR & pay a fixed rate of 5% per annum every 6 months for 3 years on a notional principal of $100 million

		---------Millions of Dollars---------		
Date	LIBOR Rate	*FLOATING* Cash Flow	*FIXED* Cash Flow	Net Cash Flow
Mar.5, 2007	4.2%			
Sept. 5, 2007	4.8%	+2.10	–2.50	–0.40
Mar.5, 2008	5.3%	+2.40	–2.50	–0.10
Sept. 5, 2008	5.5%	+2.65	–2.50	+0.15
Mar.5, 2009	5.6%	+2.75	–2.50	+0.25
Sept. 5, 2009	5.9%	+2.80	–2.50	+0.30
Mar.5, 2010	6.4%	+2.95	–2.50	+0.45

IRS Decompositions

- IRS:Cash Flows (Notional amount = 1, $\tau = \Delta t$)

TIME 0	τ	2τ	…	$(n-1)\tau$	$n\,\tau$	
Inflow		$r_0\,\tau$	$r_1\,\tau$	…	$r_{n-2}\,\tau$	$r_{n-1}\,\tau$
Outflow		$R\,\tau$	$R\,\tau$	…	$R\,\tau$	$R\,\tau$

- Decomposition 1: 2 bonds, Long Floating Rate, Short Fixed Rate

TIME 0	τ	2τ	…	$(n-1)\tau$	$n\,\tau$	
Inflow		$r_0\,\tau$	$r_1\,\tau$	…	$r_{n-2}\,\tau$	$1+r_{n-1}\,\tau$
Outflow		$R\,\tau$	$R\,\tau$	…	$R\,\tau$	$1+R\,\tau$

- Decomposition 2: n FRAs

TIME	0	τ	2τ	…	$(n-1)\tau$	$n\,\tau$
Cash flow		$(r_0 - R)\tau$	$(r_1 - R)\tau$	…	$(r_{n-2} - R)\tau$	$(r_{n-1} - R)$

Valuation of an IR swap

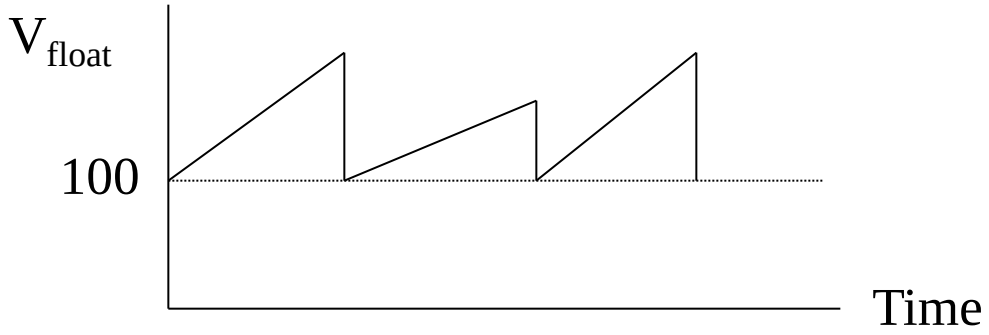
- Since a long position position of a swap is equivalent to:
  - a long position on a floating rate note
  - a short position on a fix rate note
- Value of swap (V_{swap}) equals:
  - Value of FR note V_{float} - Value of fixed rate bond V_{fix}

$$V_{swap} = V_{float} - V_{fix}$$

- Fix rate R set so that Vswap = 0

Valuation of a floating rate note

- The value of a floating rate note is equal to its face value at each payment date (ex interest).
- Assume face value = 100
- At time *n*: $V_{float,\,n} = 100$
- At time *n-1*: $V_{float,n-1} = 100\,(1+r_{n-1}\tau)/\,(1+r_{n-1}\tau) = 100$
- At time *n-2*: $V_{float,n-2} = (V_{float,n-1} + 100 r_{n-2}\tau)/\,(1+r_{n-2}\tau) = 100$
- and so on and on….



Swap Rate Calculation

- Value of swap: $f_{swap} = V_{float} - V_{fix} = M - M\,[R \Sigma d_i + d_n]$

  where d_t = discount factor
- Set R so that $f_{swap} = 0 \Rightarrow R = (1-d_n)/(\Sigma d_i)$
- Example 3-year swap - Notional principal = 100

  Spot rates (continuous)

Maturity	1	2	3
Spot rate	4.00%	4.50%	5.00%
Discount factor	0.961	0.914	0.861

- $R = (1- 0.861)/(0.961 + 0.914 + 0.861) = 5.09\%$

Swap: portfolio of FRAs

- Consider cash flow i : $M\ (r_{i-1} - R)\ \Delta t$
  - » Same as for FRA with settlement date at i-1
- Value of cash flow $i = M\ d_{i-1} - M(1 + R\Delta t)\ d_i$
- Reminder: $V_{fra} = 0$ if R_{fra} = forward rate $F_{i-1,I}$
- $V_{fra\ t-1}$
  - » > 0 If swap rate R > fwd rate $F_{t-1,t}$
  - » = 0 If swap rate R = fwd rate $F_{t-1,t}$
  - » <0 If swap rate R < fwd rate $F_{t-1,t}$
- => SWAP VALUE $= \Sigma\ V_{fra\ t}$

Other types

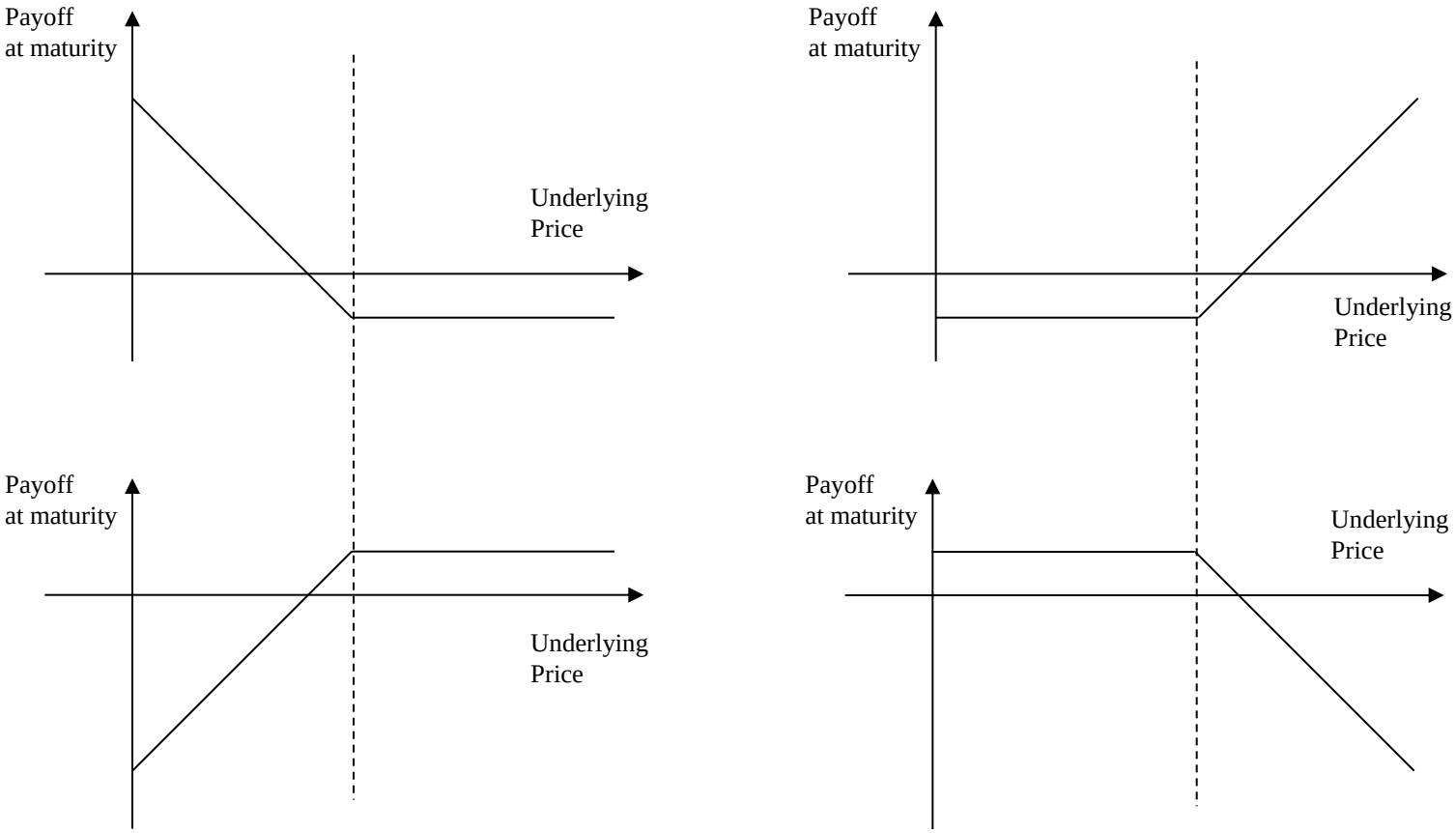
- Floating-for-floating interest rate swaps, amortizing swaps, step up swaps, forward swaps, constant maturity swaps, compounding swaps, LIBOR-in-arrears swaps, accrual swaps, diff swaps, cross currency interest rate swaps, equity swaps, extendable swaps, puttable swaps, swaptions, commodity swaps, volatility swaps……..

Options

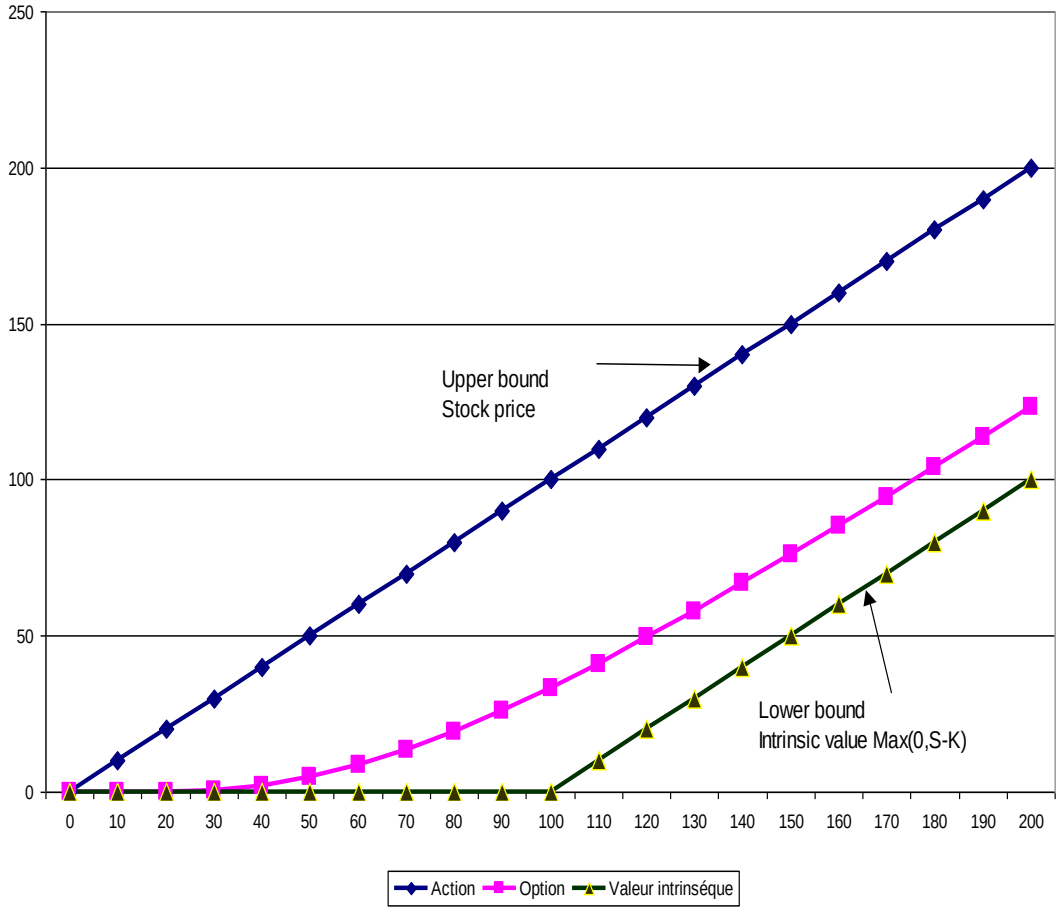
- Standard forms
  - » Call: right to buy tomorrow something at a today's fixed price
    - ✓ Buyer's payoff at maturity: $Max(S_T - K, 0) = (S_T - K)^+$
    - ✓ Value today: $e^{-rT} \mathrm{E}_0^Q (S_T - K)^+$
  - » Put: right to sell tomorrow something at a today's fixed price
    - ✓ Buyer's payoff at maturity: $Max(K - S_T, 0) = (K - S_T)^+$
    - ✓ Value today: $e^{-rT} \mathrm{E}_0^Q (K - S_T)^+$

Payoff profiles

- European Options payoff profiles at maturity



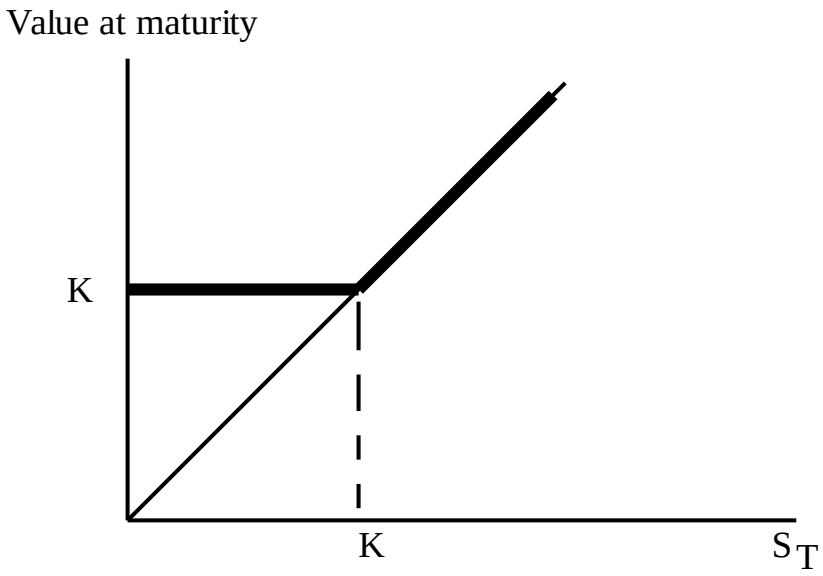
Graph of European call



Example: Insurance with a put

- Strategy 1.
  - » Buy one share + one put
- At maturity T:

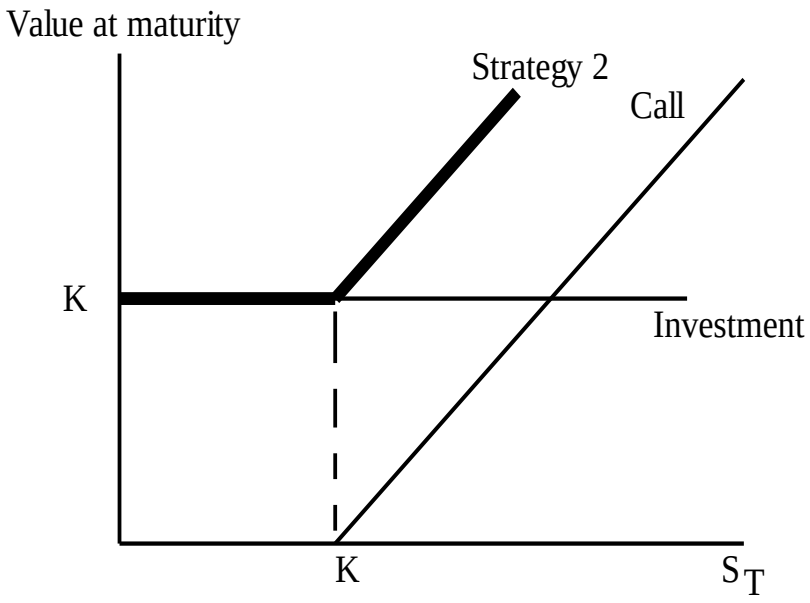
Date	$S_T<K$	$K > S_T$
Share value	S_T	S_T
Put value	$(K - S_T)$	0
Total value	K	S_T



Example: Another strategy to achieve the same result

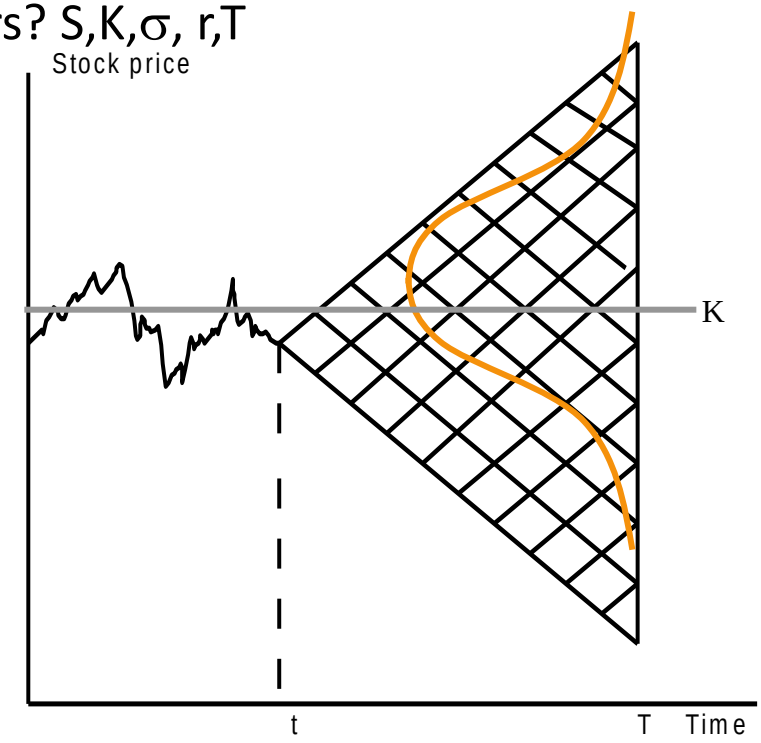
- Strategy 2
  - » Buy one call + invest PV(K)
- At maturity T:

Date	$S_T<K$	$K > S_T$
Call value	0	S_T - K
FutVal(PV(K))	K	K
Total value	K	S_T



Valuation

- Standard forms
  - » Features: American/European
  - » Pricing: Binomial/Black&Scholes/Simulations/Finite differences...
  - » Parameters? S,K,σ, r,T



Black-Scholes model

- Call price $C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$

$$d_1 = \frac{\ln\left(\frac{Se^{-qT}}{Ke^{-rT}}\right)}{\sigma\sqrt{T}} + 0.5\sigma\sqrt{T} \qquad d_2 = d_1 - \sigma\sqrt{T-t}$$

- Put price $P = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$
- Parameters
  - S =current value of underlying
  - K =strike price
  - T =time-to-maturity
  - σ =standard deviation of $\Delta S/S$
  - r =riskfree rate
  - y =dividend rate=opportunity cost of waiting, etc...
  - N(z) =cumulative standard normal probability density from $-\infty$ to z

(European) Put-Call parity

- The two strategies are equivalent in cash flows
  - » By the AOA $\Rightarrow$ should have the same cost/price

$$S_0 + P = C + \text{PV}(K)$$
$$S_0 + P = C + Ke^{-rT}$$

where
- ✓ S_0: current stock price
- ✓ P : current put value
- ✓ C : current call value
- ✓ PV(K) : present value of the strike price

- Decomposition of a European call option
  - » $C = S_0 + P - \text{PV}(K)$
  - » Buying a European call is equivalent to:
    - ✓ buying of stock
    - ✓ buying a put (insurance)
    - ✓ borrowing the PV(K) (leverage)
- Decomposition of a forward
  - » $C - P = S - \text{PV}(K) = \textit{Forward}$

References

- Hull slides
- BIS, Statistics, http://www.bis.org/statistics
- OCC Quarterly Derivatives Fact Sheets: http://www.occ.treas.gov/deriv/deriv.htm